Feshbach Projection Operator Partitioning for Quantum Open Systems: Stochastic Approach

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Dynamics of a state of interest coupled to a non-Markovian environment is studied for the first time by concatenating the non-Markovian quantum state diffusion (QSD) equation and the Feshbach projection operator partitioning technique. An exact one-dimensional stochastic master equation is derived as a general tool for controlling an arbitrary component of the system. We show that the exact one-dimensional stochastic master equation can be efficiently solved beyond the widely adapted second-order master equations. The generality and applicability of this hybrid approach is justified and exemplified by several coherence control problems concerning quantum state protection against leakage and decoherence.

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I. INTRODUCTION

Feshbach projection operator partitioning technique (termed PQ partitioning for brevity) allows one to focus exclusively on the dynamics of a small subspace in the whole Hilbert space of an open or closed quantum system [1, 2]. This has been proven to be exceptionally useful in dealing with the decoherence suppression for quantum states within many interesting contexts in physics such as quantum state storage where PQ partitioning can significantly reduce the resource by focusing on a target state rather than on the entire multi-level (even infinite level) Hilbert space. The targeted component in the Hilbert space, called P subspace, could be a superposition of some or all of the energy levels, while the rest of the state space is denoted as Q subspace. Generally there is a bidirectional wave-function flow between P and Q subspaces, unless some intervention mechanism is implemented. A typical example of this case is the Quantum Zeno Effect, that may decompose the whole space of the system into isolated Zeno subspaces [3, 4] and the remaining parts of the whole space. When the system is embedded in a dissipative environment, the dissipation of the whole system and the leakage of the subspace will mix with each other to leave a compelling, yet subtle question of the dynamical control of one subspace in open quantum systems, especially in those with a non-Markovian environment.

A majority of prior research efforts on a non-Markovian environment are based on a second-order master equation in terms of coupling constant obtained by using projection-operator technique [5] or the Liouvillean approach [6], where the weak coupling regime is typically assumed. The work presented in this paper will investigate the dynamics and control of an arbitrary subspace of an open system beyond the weak non-Markovian regime by employing the non-Markovian quantum state diffusion equation [7–11], which is capable of dealing with the strong coupling strength and the arbitrary correlation function of the environment. The non-Markovian QSD equation may be cast into a convolutionless form, hence it also serves as a useful tool in deriving the corresponding exact master equation [10, 12, 13]. In addition, the approximate QSD equation obtained by perturbation may also include the contributions from the high-order non-Markovian master equation [14–16].

Based on a microscopic quantum dissipative model, we will first derive a general one-dimensional dynamical equation for a subspace by combining the QSD equation and the PQ partitioning technique. The system survival probability in the subspace is obtained by the ensemble average over the trajectories for the survival amplitude. For simplicity but without loss of generality, we consider an initial pure state as our subspace of interest (with dimension one).

We show in this paper via the stochastic approach that the non-Markovian dynamics of the one dimensional subspace can be effectively controlled by a sequence of pulses applied to the system. In particularly, we show that the control function $c(t)$ can be chosen as a simple periodic rectangular interaction [17]: $c(t)=\frac{\Psi}{\Delta}$, where Ψ is the interaction intensity, for regions $n\tau-\Delta<t<n\tau$, $n\geqslant 1$ integral; otherwise $c(t)=0$, where $1/\tau$ is the frequency of the pulse and Δ is the duration time of the pulse in one period. The pulses applied here are neither an ideal zero-width pulse nor an optimized Bang-Bang control [18]. Yet it is sufficient for our control scheme if the interval τ is short enough. We find that increasing Ψ is also beneficial to reduce the leakage rate of the subspace.

II. FESHBACH PQ PARTITIONING AND ONE-DIMENSIONAL STOCHASTIC MASTER EQUATION

We consider an open quantum system with $(N+1)$ normalized base vectors (N is arbitrary) coupled to a bath of harmonic oscillators described by the following total Hamiltonian (setting $\hbar = 1$):

$$H_{\rm tot} = H_{\rm sys} + \sum_\lambda (g^*_\lambda L a^\dagger_\lambda + g_\lambda L^\dagger a_\lambda) + \sum_\lambda \omega_\lambda a^\dagger_\lambda a_\lambda, \quad (1)$$

where $H_{\rm sys}$ and L are the system Hamiltonian and the Lindblad operator, respectively. The system state described by a stochastic wave-function ψ_t is governed by the non-Markovian QSD equation [7, 8]:

$$i\partial_t \psi_t = \left[H_{\rm sys} + iLz^*_t - iL^\dagger \bar{O}(t,z^*)\right]\psi_t = H_{\rm eff}\psi_t. \quad (2)$$

Here z^*_t is the colored noise arising from coupling to the environment such that its statistical mean recovers the environment correlation function $\alpha(t,s)$: $M[z_t z^*_s] = \alpha(t,s)$. Note that $\bar{O}(t,z^*)$, with its explicit expression given below, is the system operator representing the effect of the environment. Thus the effective Hamiltonian $H_{\rm eff}$ contains all the information about the open system and its interaction with the environment. Whenever the operator $\bar{O}$ is exactly constructed, then effective Hamiltonian is exact. That is, it is directly derived from the total Hamiltonian without using any approximations, in particular, without Born-Markov approximation.

The PQ partitioning technique can divide the system wave-function ψ_t (with $(N+1)$-dimension) into two parts: a scalar function $P(t)$ associated with a chosen vector denoted by $|0\rangle$ and an N-dimensional vector $Q(t)$. Note that $|0\rangle$ can be an arbitrary component of the system and doest not necessarily denote the ground state. With this partition, the state and the effective Hamiltonian may be written as

$$\psi_t = \left[\begin{array}{c} P \\ \hline Q \end{array}\right], \quad H_{\rm eff} = \left(\begin{array}{c|c} h & R \\ \hline W & D \end{array}\right). \quad (3)$$

Here the 1×1-matrix h and the $N\times N$-matrix D correspond to the self-Hamiltonians living in the P subspace and the Q subspace, respectively. For the effective Hamiltonian, the resulting W and R are not mutually conjugate to each other in general. Consequently, the QSD equation (2) may be decomposed into two parts:

$$i\partial_t P = hP + RQ, \quad (4)$$
$$i\partial_t Q = WP + DQ. \quad (5)$$

The solution for Eq. (5) could be formally expressed by

$$Q(t) = -i\int_0^t ds G(t,s)W(s)P(s) + G(t,0)Q(0), \quad (6)$$

where $\partial_t G(t,s) = -iD(t)G(t,s)$ and $G(t,t) = 1$. Note that the propagator may be written as

$$G(t,s) = \mathcal{T}_\leftarrow \left\{\exp\left[-i\int_s^t D(s')ds'\right]\right\}, \quad (7)$$

where $\mathcal{T}_\leftarrow$ is the time-ordering operator. Substituting Eq. (6) into Eq. (4), we obtain a closed one-dimensional master equation for $P(t)$:

$$\begin{aligned} i\partial_t P(t) &= h(t)P(t) - i\int_0^t ds\tilde{G}(t,s)P(s) \\ &\quad + R(t)G(t,0)Q(0), \\ \tilde{G}(t,s) &= R(t)G(t,s)W(s). \end{aligned} \quad (8)$$

In case that matrix D is diagonalizable or $G(t,s) \approx \sum_{n=1}^N e^{-i\int_s^t D_{nn}(s')ds'}|n\rangle\langle n|$ is a good approximation, Eq. (8) can be readily calculated. Especially when $\tilde{G}(t,s) = 0$, a formal solution can be given,

$$P(t) = \left[P(0) - i\int_0^t ds' R(s')Q(s')e^{i\int_0^{s'} ds h(s)}\right]e^{-i\int_0^t ds h(s)}. \quad (9)$$

Notably, the off-diagonal term $R(t)$ between P and Q plays a vital role in the dynamics of $P(t)$ while the local control term $h(t)$ only provides a phase factor.

Equation (8) will be the main result in the concatenation of the PQ partitioning and the non-Markovian QSD equation described by $H_{\rm eff}$. As shown below, the one-dimensional master equation (8) can yield some very interesting analytical results that are hard to obtain without invoking certain approximations such as weak-coupling approximation. We will now discuss three examples, where $\bar{O}(t,z^*)$ is exact, hence $H_{\rm eff}$ is exact and time-local, to illustrate both the free and controlled dynamics of states of interest coupled to a zero temperature heat bath, in terms of their fidelity with respect to the initial state. For an arbitrary pure initial state ψ_0, it is easily to show that the fidelity $\mathcal{F}(t) = M[|P(0)^*P(t) + Q^\dagger(0)Q(t)|^2]$. A simpler expression for the fidelity is obtained $\mathcal{F}(t) = M[|P(t)|^2]$ when $Q(0) = \mathbf{0}$, which is equivalent to the population or survival probability over ψ_0. Remarkably, in many physically interesting examples as shown below, the PQ partitioning combined with the QSD equation can yield simple analytical solutions of the fidelity $\mathcal{F}(t)$.

A. Dissipative two-level atom model

The model is represented by $H_{\rm sys} = E_0(t)|0\rangle\langle 0| + E_1(t)|1\rangle\langle 1|$ and $L = |0\rangle\langle 1|$. The effective Hamiltonian in the basis $|0\rangle, |1\rangle$ becomes

$$H_{\rm eff} = \left(\begin{array}{c|c} E_0(t) & iz^*_t \\ \hline 0 & E_1(t) - iF(t) \end{array}\right), \quad (10)$$

where $F(t)$ is the coefficient function in the operator $\bar{O}(t,z^*) = F(t)L$ with $F(t) \equiv \int_0^t ds\alpha(t,s)f(t,s)$ and the initial condition $F(0) = 0$. The equation of motion for $f(t,s)$ is given by $\partial_t f(t,s) = [iE(t) + F(t)]f(t,s)$ [8], where $E(t) = E_1(t) - E_0(t)$. For this two-level atom and the systems in the following two examples, we have

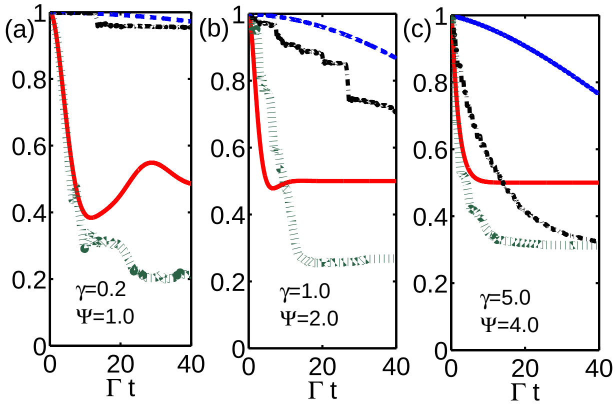

FIG. 1. Fidelity dynamics of the two-level system with different γ and Ψ. Red solid lines for free dynamics, blue dashed lines for $\tau = 2\Delta$, black dot-dashed lines for $\tau = 3\Delta$, and green dotted lines for $\tau = 6\Delta$. The other parameters are set as $\omega = 0.2\Gamma$, and $\Delta = 0.04\Gamma t$.

$E(t) = \omega + c(t)$, where ω is the bare frequency for the system and $c(t)$ is the control function.

With the initial state of the system $|\psi_0\rangle = (1/\sqrt{2})(|0\rangle + |1\rangle)$, our aim is to control the population of this chosen state. The exact stochastic transition amplitude is

$$\langle\psi_0|\psi_t\rangle = \frac{1}{2}\Big\{ e^{-i\int_0^t dsE_1'(s)} + e^{-i\int_0^t dsE_0(s)} \times \Big[1 + \int_0^t ds' z_{s'}^* e^{-i\int_0^{s'} ds[E_1'(s)-E_0(s)]}\Big]\Big\}, \quad (11)$$

where $E_1'(s) \equiv E_1(s) - iF(s)$. Consequently in the rotating picture of $H_{\rm sys}$, we have

$$\mathcal{F}(t) = \frac{1}{4}\Big[1 + \bar{F}_R^2(t) + 2\bar{F}_R(t)\bar{F}_I(t) + \int_0^t\int_0^t ds_1 ds_2 \times \alpha(s_1,s_2)\bar{F}_R(s_1)\bar{F}_R(s_2)\bar{K}(s_1)/\bar{K}(s_2)\Big], \quad (12)$$

where $\bar{F}_R(t) \equiv e^{-\int_0^t dsF_R(s)}$, $\bar{F}_I(t) \equiv \cos[\int_0^t dsF_I(s)]$ (F_R and F_I stand for the real and imaginary parts of F respectively) and $\bar{K}(t) \equiv e^{i\int_0^t ds[E(s)+F_I(s)]}$.

For simplicity, throughout the paper the non-Markovian environmental noise is described by an Ornstein-Uhlenbeck type correlation [19] $\alpha(t,s) = \frac{\Gamma\gamma}{2}e^{-\gamma|t-s|}$, where Γ is the environmental dissipation rate and $1/\gamma$ characterizes the memory time of the environment. The finite, but non-zero $1/\gamma$ gives a non-Markovian process with the Markov limit when $\gamma \to \infty$. The equation of motion for $F(t)$ can be easily derived from the QSD equation:

$$\partial_t F(t) = \frac{\Gamma\gamma}{2} + [-\gamma + iE(t)]F(t) + F^2(t). \quad (13)$$

Therefore, Eq. (12) may be controled by the functions $E(t)$ (or $c(t)$) via Eq. (13).

When $c(t) = 0$, the two-level atom will be driven by the environment into the ground state after a period of time. The red lines in Fig.1 show that the fidelity approaches 0.5 regardless of the memory times. Clearly, in the absence of control function, the smaller γ typically gives rise to more significant fluctuations, yet it does not effectively preserve the fidelity. However, when the control pulses are applied, we see that the non-Markovian features can greatly enhance the efficiency of the rectangular pulses for controlling the state of interest. The blue dashed line in Fig.1(a) indicates that the fidelity can be preserved for a long period of time. Particularly, when the period of the pulse τ is chosen as small as twice the interaction duration time Δ in one period, the fidelity is fully preserved up to the time $\Gamma t = 13$. Moreover, we show that the control pulses can work very well even if τ is as long as the three times of Δ (the black dot-dashed line). But if the period of pulses is too long, the dynamical decoupling becomes highly inefficient [20]. For example, when $\tau = 6\Delta$, as shown by the green dotted lines in Figs.1(a), 1(b) and 1(c), the control function $c(t)$ becomes adversary of the fidelity control. In all those three cases, the fidelity decays faster than those of free evolution (*i.e.*, $c(t) = 0$). With decreasing memory time of the environment plotted in Figs.1(b) and 1(c), the control effect of the fidelity is gradually weakened even using the same rapid pulses as in Fig.1(a) and simultaneously raising the interaction intensity Ψ for partial compensation. When $\gamma = 2.0$ and $\tau \geqslant 3\Delta$, we see that the control becomes inefficient in the Markov limit (large γ).

B. A qutrit dissipative model

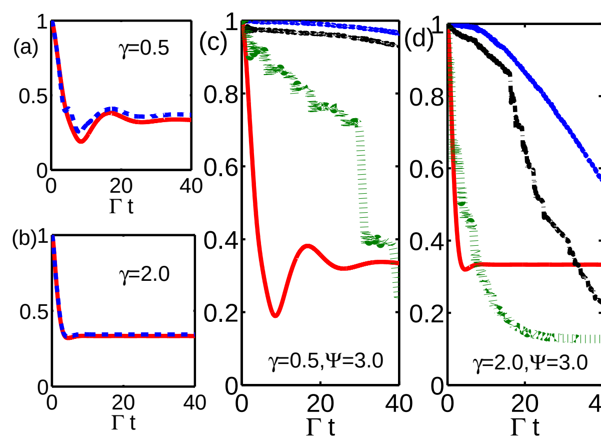

FIG. 2. Fidelity dynamics of the qutrit system with different γ. In (a) and (b), the approximate results (blue dashed lines) by Eq. (14) is compared with the exact ones (red solid lines); In (c) and (d), Red solid lines for free dynamics, blue dashed lines for $\tau = 2\Delta$, black dot-dashed lines for $\tau = 3\Delta$, and green dotted lines for $\tau = 6\Delta$. The other parameters are set as $\omega = 1.0\Gamma$, $\Delta = 0.04\Gamma t$, and $\kappa = \sqrt{2}$.

For this model, we have $H_{\rm sys} = E(t)(|2\rangle\langle 2| - |0\rangle\langle 0|)$ and $L = \kappa(|0\rangle\langle 1| + |1\rangle\langle 2|)$. In the basis $|1\rangle$, $|0\rangle$ and $|2\rangle$, the effective Hamiltonian takes for the following form:

$$H_{\rm eff} = \left(\begin{array}{c|cc} -iF_1 & 0 & i\kappa z_t^* - i\kappa U_z \\ \hline i\kappa z_t^* & -E & 0 \\ 0 & 0 & E - iF_2 \end{array}\right), \qquad (14)$$

where $F_1(t)$, $F_2(t)$ and $U_z(t) = \int_0^t ds' U(t,s') z_{s'}^*$ are the three coefficient functions in $\bar{O}(t, z^*) = F_1(t)|0\rangle\langle 1| + F_2(t)|1\rangle\langle 2| + U_z(t)|0\rangle\langle 2|$. One particularly interesting feature of this model is that in the PQ partitioning $W \neq 0$, but D is diagonal, so it allows a simple solution. Applying the noise-free approximation, we have $\bar{O}(t, z^*) = F_1(t)|0\rangle\langle 1| + F_2(t)|1\rangle\langle 2|$ with

$$\frac{dF_1(t)}{dt} = \frac{\Gamma\gamma\kappa^2}{2} + (-\gamma + iE)F_1 + F_1^2 - F_1F_2,$$
$$\frac{dF_2(t)}{dt} = \frac{\Gamma\gamma\kappa^2}{2} + (-\gamma + iE)F_2 + F_2^2, \qquad (15)$$

and $F_j(0) = 0$, $j = 1, 2$. The approximation validity in Eq. (14) is testified by Fig.2(a) and Fig.2(b) with two different γ's. We could also find the analytical expression of the fidelity by PQ partitioning technique. When $|\psi_0\rangle = (1/\sqrt{3})(|0\rangle + |1\rangle + |2\rangle)$, the compact formula for the fidelity is given by

$$\begin{aligned}\mathcal{F}(t) = \frac{1}{9}\Big\{ & |1 + \bar{F}_1(t) + \bar{F}_2(t)|^2 + \int_0^t ds \int_0^t ds_1 \alpha(s_1, s) \\ & \times [\bar{F}_1^*(t)\bar{B}^*(s_1) + \bar{F}_1^*(s_1)\bar{E}^*(s_1)][\bar{F}_1^*(t)\bar{B}^*(s_1) \\ & + \bar{F}_1^*(s_1)\bar{E}^*(s_1)] + \int_0^t ds \int_0^s ds' \int_0^t ds_1 \int_0^{s_1} ds_2 \\ & \times [\alpha(s_1, s)\alpha(s_2, s') + \alpha(s_1, s')\alpha(s_2, s)] \\ & \times \bar{F}_1^*(s_1)\bar{E}^*(s_1)\bar{B}^*(s_2)\bar{F}_1(s)\bar{E}(s)\bar{B}(s')\Big\}, \end{aligned} \qquad (16)$$

where $\bar{F}_j(t) \equiv e^{-\int_0^t ds F_j(s)}$, $j = 1, 2$, $\bar{E}(t) \equiv e^{-i\int_0^t ds E(s)}$, and $\bar{B}(t) \equiv \bar{E}(t)\bar{F}_2(t)/\bar{F}_1(t)$.

In Figs.2(c) and 2(d), we compare the control dynamics with the same environments as in Figs.2(a) and 2(b) respectively. As in the first example, we also notice the larger environmental memory time (less γ) is very helpful to relieve the requirement of the pulse frequency than the smaller one (bigger γ). If $\gamma = 0.5$, $\tau \leqslant 3\Delta$, the fidelity could be maintained above 0.9 even when Γt approaches 40. In Fig.2(d), $\gamma = 2.0$, it is interesting to observe a damping enhancement of the fidelity especially when $\tau \geqslant 3\Delta$. This (as well as all the green dotted lines in Fig.1) corresponds to a sort of Quantum Anti-Zeno Effect (AZE) [21]. AZE occurs when the evolution is repetitively interrupted by projecting the system onto some state as in a measurement process [22], by periodically applying the pulses [23] with long time intervals, or by coupling between the excited state and an auxiliary state [24]. Here it is induced by unitary pulse sequences, which renormalize the frequency of the system with a proper period and mimic a repeated measurement process.

C. A special $(N+1)$-level atom

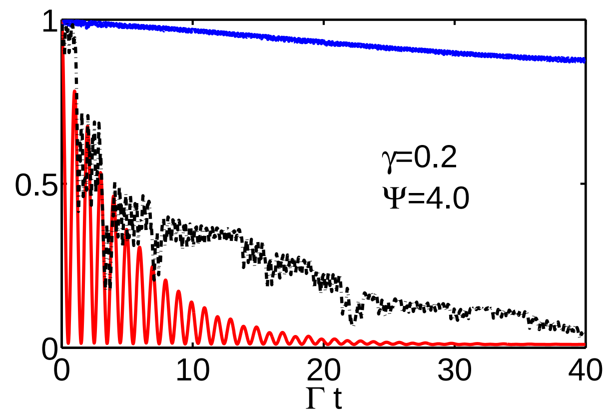

FIG. 3. Fidelity dynamics of a 101-level system in a non-Markovian environment. Red solid line for free dynamics, blue dashed line for $\tau = 2\Delta$, and black dot-dashed lines for $\tau = 3\Delta$. The other parameters are set as $\omega = 0.2\Gamma$, $\Delta = 0.04\Gamma t$.

For this special case, we consider a genuine multi-level atomic system represented by $H_{\rm sys} = \sum_{n=0}^N E_n|n\rangle\langle n|$ and $L = \sum_{n=1}^N |0\rangle\langle n|$, where $E_0 = -E(t)$ and $E_{n\neq 0} = E(t)$, and for this model $\bar{O}(t, z^*) = F(t)L$. Therefore,

$$H_{\rm eff} = \left(\begin{array}{c|cccc} -E & iz_t^* & iz_t^* & \cdots & iz_t^* \\ \hline 0 & E - iF & -iF & \cdots & -iF \\ 0 & -iF & E - iF & \cdots & -iF \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & -iF & -iF & \cdots & E - iF \end{array}\right), \qquad (17)$$

where $F(t)$ satisfies $\partial_t F(t) = \frac{\Gamma\gamma}{2} + (-\gamma + 2iE)F + NF^2$ and $F(0) = 0$. Due to the time symmetry of D, $[D(t), D(s)] = 0$ $(t \neq s)$, the propagator in Eq. (7) can be exactly obtained as

$$\begin{aligned} G(t, 0) = \frac{1}{N}\Big\{ & \sum_{j=1}^N \left[\bar{E}(t)\bar{F}^N(t) + (N-1)\bar{E}(t)\right]|j\rangle\langle j| \\ & + \sum_{n\neq m} [\bar{E}(t)\bar{F}^N(t) - \bar{E}(t)]|n\rangle\langle m|\Big\}, \end{aligned} \qquad (18)$$

where $\bar{F}(t) \equiv e^{-\int_0^t F(s)ds}$. For the initial state $|\psi_0\rangle = (1/\sqrt{N+1})\sum_{n=0}^N |n\rangle$, again the PQ partitioning allows to find an analytical expression for the fidelity function:

$$\begin{aligned}\mathcal{F}(t) = \frac{1}{(N+1)^2}\Big[& 1 + N^2\bar{F}_R^{2N}(t) + 2N\bar{F}_R^N(t)\bar{F}_I'(t) + N^2 \\ & \times \int_0^t \int_0^t ds_1 ds_2 \alpha(s_1, s_2)\bar{F}_R^N(s_1)\bar{F}_R^N(s_2)\bar{K}'(s_1)/\bar{K}'(s_2)\Big], \end{aligned} \qquad (19)$$

where $\bar{F}_I'(t) \equiv \cos[\int_0^t ds N I_F(s)]$ and $\bar{K}'(t) \equiv e^{i\int_0^t ds[2E(s) + NI_F(s)]}$. It is easy to check that when $N = 1$,

Eq. (19) reduces to Eq. (12). It is interesting to consider the weak coupling limit when $F \to 0$, then D becomes a diagonal matrix diag$[E, E, \cdots, E]$, thus Eq. (7) can be simplified as

$$\mathcal{F}(t) = \frac{1}{(N+1)^2}\Bigg[1 + N^2 + 2N + N^2 \int_0^t \int_0^t ds_1 ds_2 \times \alpha(s_1, s_2)\bar{E}^2(s_2)/\bar{E}^2(s_1)\Bigg]. \quad (20)$$

We emphasize that our formalism works for an arbitrary $N+1$ system. For example, we take $N = 100$ shown in Fig.3. Without the control function $c(t)$, the fidelity will quickly decays to $1/(N+1)$. Clearly, the efficient control can be made possible only when the system is far from Markov regime. As shown in this example, our pulse control scheme works well for the non-Markovian environment with $\gamma = 0.2$. In fact, by setting $\Psi = 4.0$, and $\tau = 2\Delta$, we show that $\mathcal{F}(\Gamma t = 40)$ can be maintained as high as 0.85.

III. CONCLUSION

We have derived an exact one-dimensional stochastic master equation based on the Feshbach PQ partitioning and the non-Markovian QSD equation and apply it to the quantum control dynamics of three distinct model systems. The periodical rectangular pulses are used to protect the subspace of interest from leakage to the other part of the Hilbert space of the system and the environment. The control dynamics measured by time-dependent fidelity is realized by tuning the pulse frequency $1/\tau$ with different memory time $1/\gamma$ of the non-Markovian environment. Our results have exemplified the simplicity and power of the exact one-dimensional stochastic master equation. Our approach has paved a way for studying the direct control dynamics of an arbitrary multi-level atomic system without invoking the exact or perturbative non-Markovian master equations. Our hybrid technique would be versatile enough to accommodate other types of quantum control if some necessary modifications on the non-Markovian QSD equations are made. It is also possible to consider the quantum control dynamics of multi-particle system [25] based on the PQ partitioning. We leave these open questions for the future investigations.

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